기계융합공학과

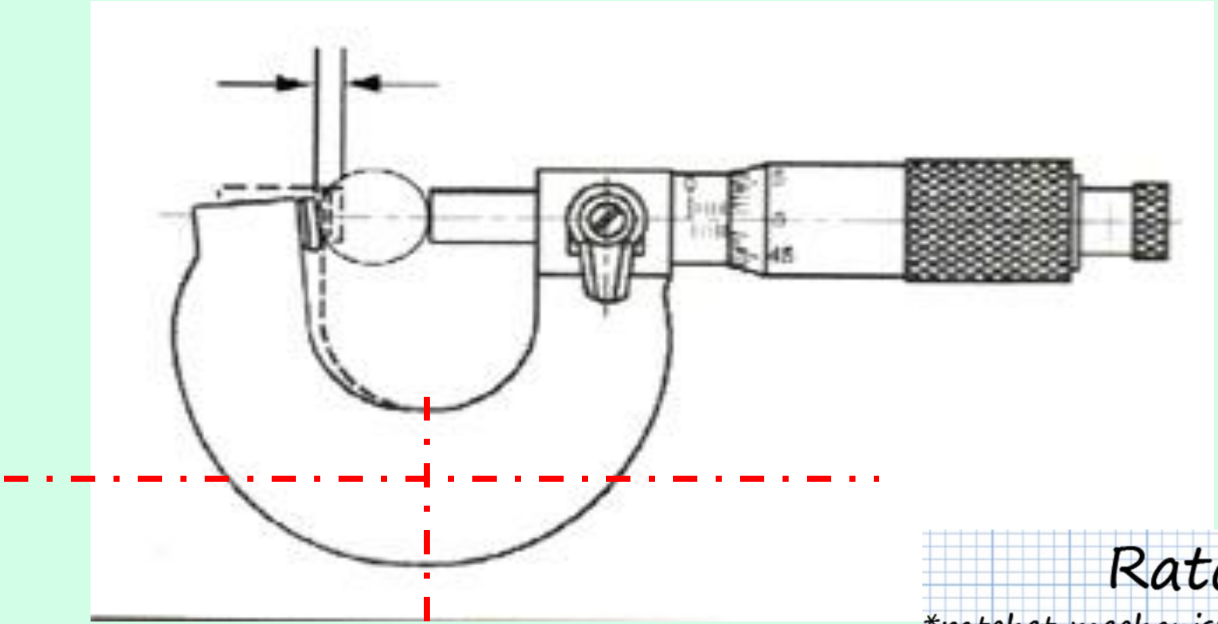
정밀공학 (Precision Engineering)

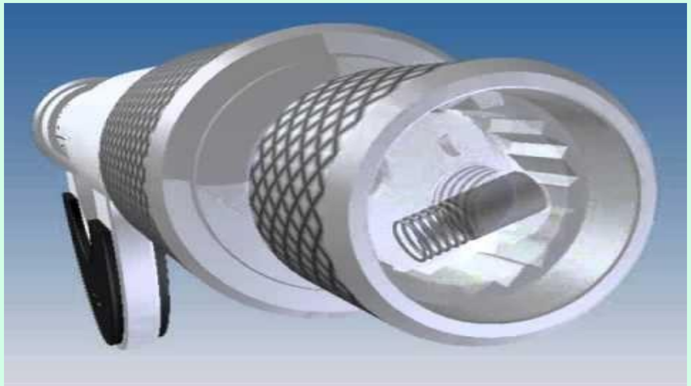
제 6 장 연성의 원리

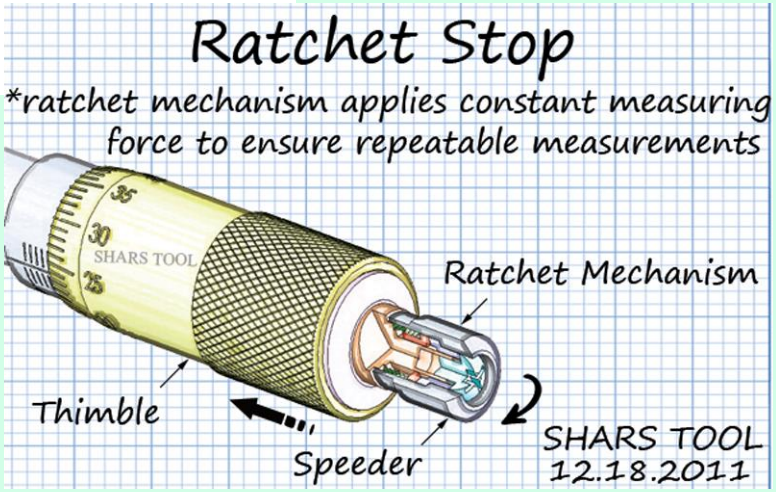
제 6 장 수업내용

- 연성의 원리(the principle of compliance)
  - 강성(stiffness)과 연성(compliance)
  - 연성의 원리
  - 연성의 원리 적용 예
    - 연성 최소화와 단면(cross section) 이용률
    - 역선(length of the line of force)의 최소화
    - 예압에 의한 연성의 최소화

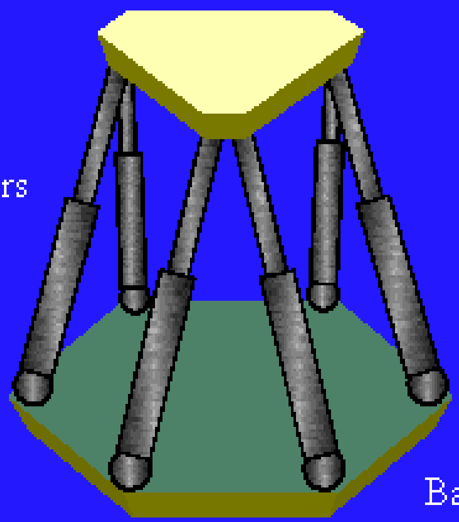
No Abbe Error?

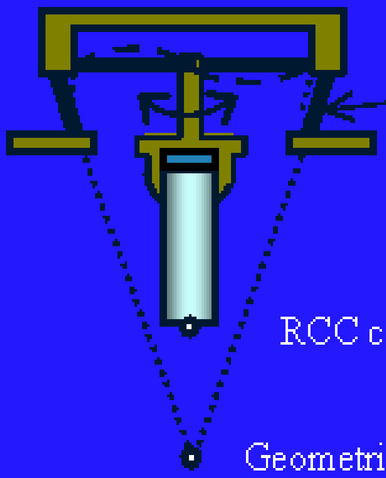






Stiffness vs Compliance





A generic Stewart Platform device. An example of high stiffness parallel mechanism. Good positional accuracy due to high stiffness.

A Remote-Center-of-Compliance (RCC) device. An example of a compliant parallel mechanism. Low stiffness facilitates insertion and assembly tasks.

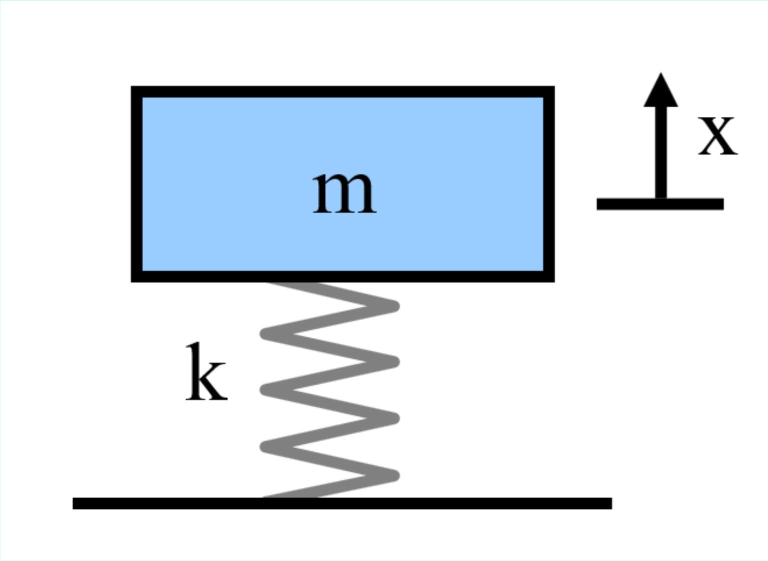
Stiffness vs Compliance

Stiffness vs Compliance

Stiffness vs Compliance

강성(stiffness)과 연성(compliance)

- 정적강성(static stiffness) vs 동적강성(dynamic stiffness)



$$M\ddot{x} + kx = 1$$

$$x_{static} = \frac{1}{k}$$

vs

$$M\ddot{x} + kx = F(t)$$

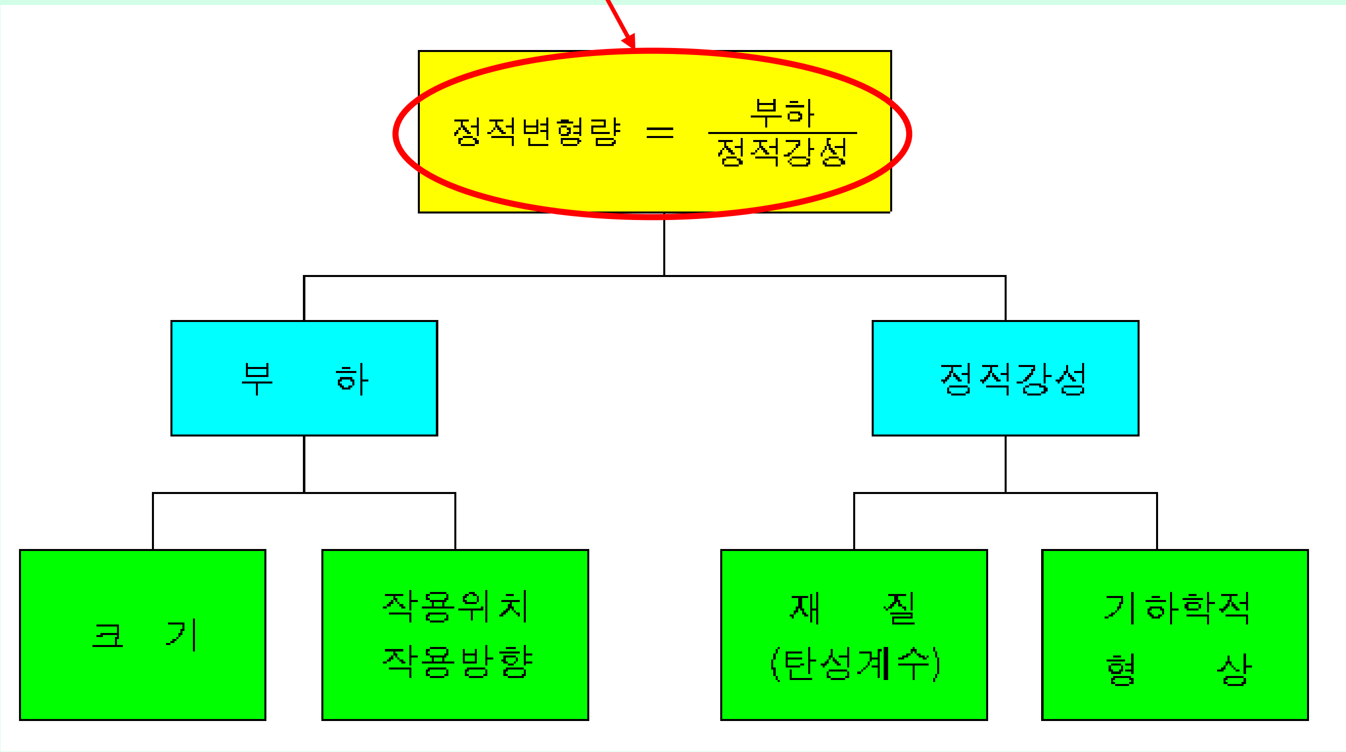
$$x = \frac{F_0}{k}[1 - cos(\omega t)]$$

단순계산으로도 2배!!!

강성(stiffness)과 연성(compliance)

❑ 강성(k)의 정의

- $k = F$(힘)/δ(변형) ➔ $\delta = (1/k)\cdot F$; 시스템의 입출력 관점에서의 함수식
- 강성을 지배하는 인자



강성(stiffness)과 연성(compliance)

❑ 정적강성(static stiffness)

- 각 구성요소 및 접합부의 정적강성을 직렬 결합한 결과
  - ➔ 정적강성이 낮은 부분의 특성이 전체적인 정적강성을 좌우
  - ➔ 기계 각부의 정적강성의 크기를 균등히 배분하는 것이 필요

❑ 동적강성(dynamic stiffness)

- 동적인 강성은 변동하는 외력(외란진동, 요소의 불평등에 의한 진동, 절삭프로세스로부터 오는 진동 등)에 주파수응답
- 1 자유도계의 진동계라고 생각하면 공진진폭 A는

$$A = \frac{F}{2\zeta K},$$ F : 변동력이고, ζ: 계의 감쇠비(≈0.01), K: 정적강성

➔ 공진 시의 변형량 : 정적 변형량의 수 배~수십 배

강성(stiffness)과 연성(compliance)

❑ 연성(c)의 정의

- 입력은 F, 출력은 δ로 전달함수가 강성의 역수형태
  ➔ 설계시스템에 사용하는 개념으로는 적당하지 않다.
- $\delta = (1/k) \cdot F \Rightarrow \delta = c \cdot F$

❑ 굽힘 연성과 비틀림 연성

▶ bending : $\dfrac{d^2\delta}{dx^2} = \dfrac{M_b}{EI}$

▶ torsion : $\dfrac{\theta}{L} = \dfrac{M_t}{GI_p}$

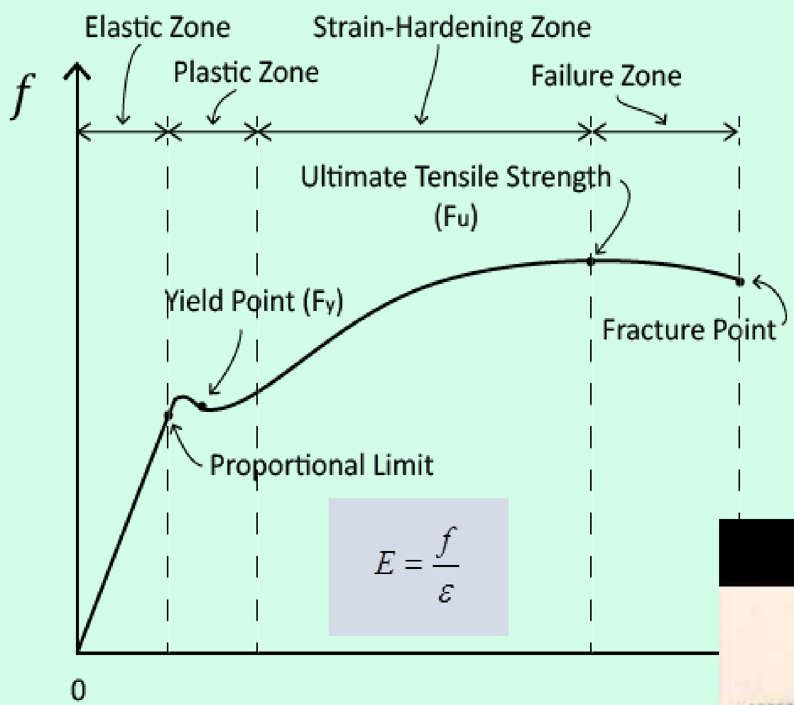
E: Young's modulus, I: 2nd moment of area

G: shear modulus, I_p: polar moment of inertia of cross-sectional area

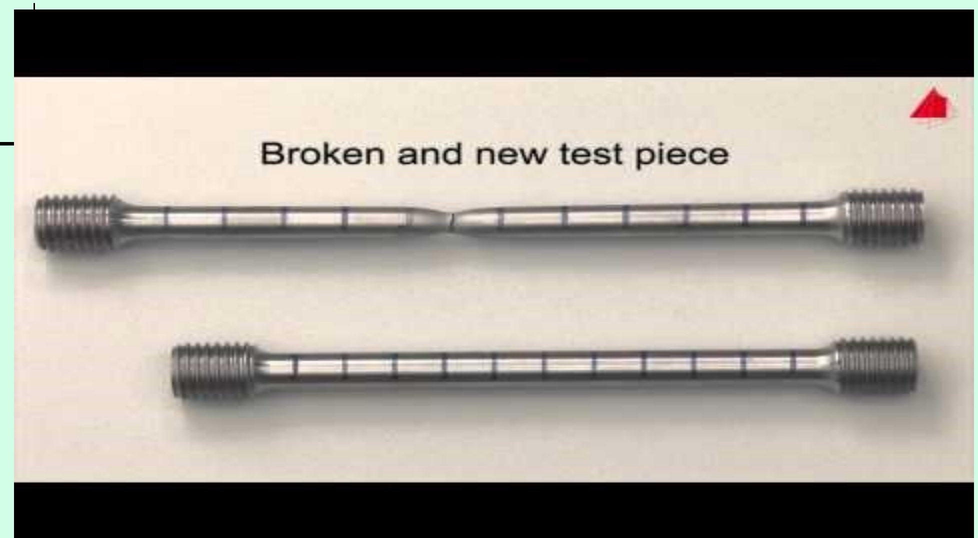
Max. B.M.?

- 굽힘연성(bending compliance): $c_b \propto 1/EI$
- 비틀림연성(torsional compliance) : $c_t \propto 1/GI_p$

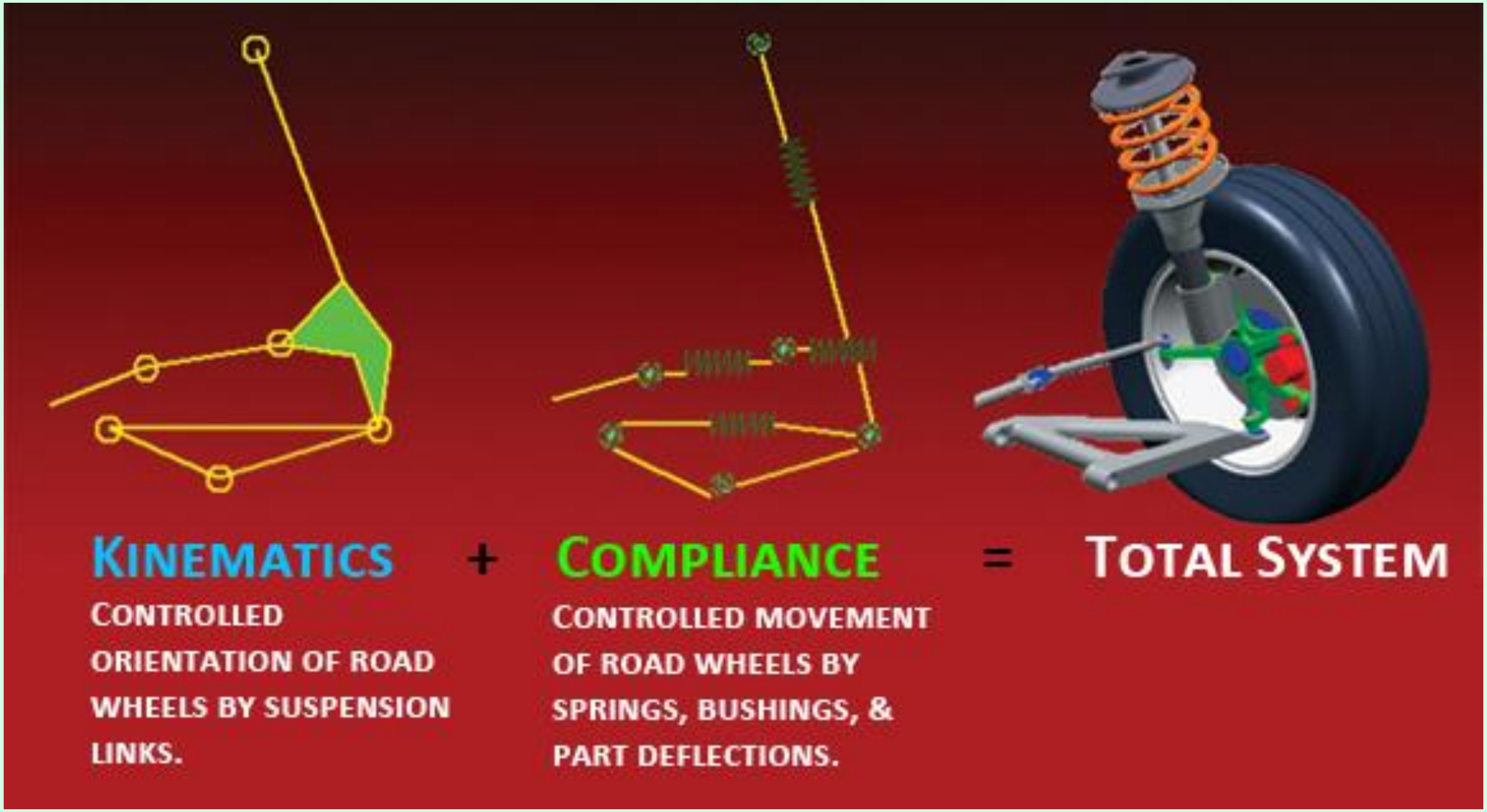
Remind Tensile Test!!



$$E = \frac{f}{\varepsilon}$$



Kinematics & Compliance



Kinematics & Compliance

연성의 원리(Principle of Compliance)

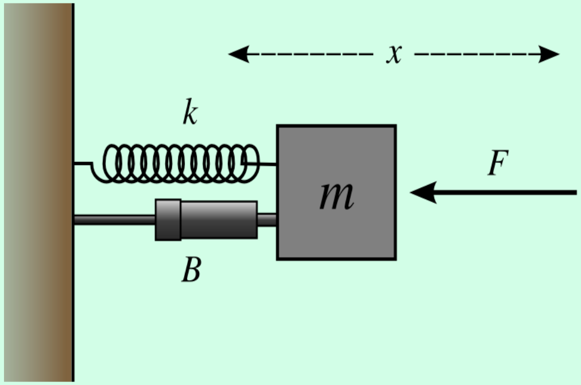
"고정도의 기계를 실현하기 위해서는 연성(compliance)을 가능한 한 작게 하는 것이 좋다."

- 영률/전단탄성률이 큰 재료 사용
- 단면이용률 향상
- 역선의 최소화
- 예압

연성 최소화와 단면이용률

- **연성감소 대책 : 탄성계수(E) 또는 전단탄성률(G)가 큰 재료 사용**
  - 취성(Brittlness)이 강한 재료는 충격에 약하므로 사용에 제한
  - 연성이 작더라도 동적으로 불안정한(감쇄계수가 작은) 경우 사용 제한
  - 복합재료의 사용이 적합한 경우가 많다.

    에폭시-그라나이트, 레진 콘크리이트, 충진재, 접합재 등
  - 재료의 변경은 가격, 가공 및 목적하는 특성의 상승에 비하여 다른 특성의 약화를 가져오기 쉬우므로 신중하게 선택



상용재료의 영률, 전단탄성률

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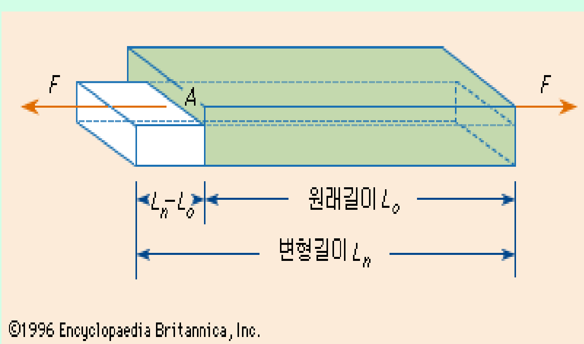

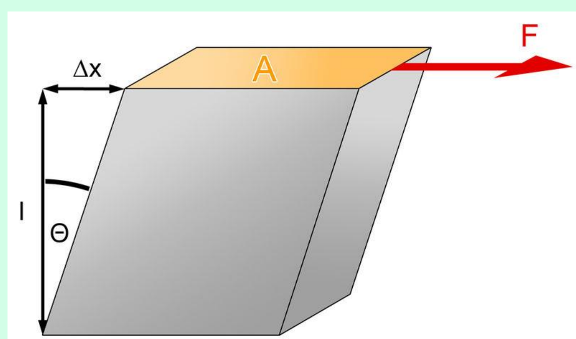

$$E \stackrel{\text{def}}{=} \frac{\sigma}{\varepsilon} = \frac{F/A_0}{\Delta L/L_0} = \frac{FL_0}{A_0 \Delta L}$$

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연성 최소화와 단면이용률

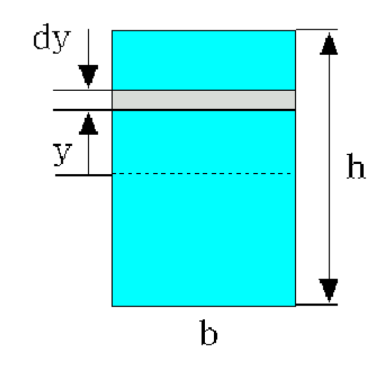
❑ 연성감소 대책 : 단면의 이용률을 향상시킨다

- 형상 증가 ⇒ 연성 감소 ⇒ 부재 하중의 증가(변형, 비용, 조립문제)
- 단면적은 감소하면서 I, I_p의 증가를 도모할 필요성
- 단면 2차 모멘트(2nd moment of area): I

$$I = \int y^2 dA$$

ex) 사각단면의 경우

$$I = 2\int_0^{\frac{h}{2}} y^2 b\,dy = 2b\left[\frac{y^2}{3}\right]_0^{h/2} = \frac{1}{12}bh^3$$



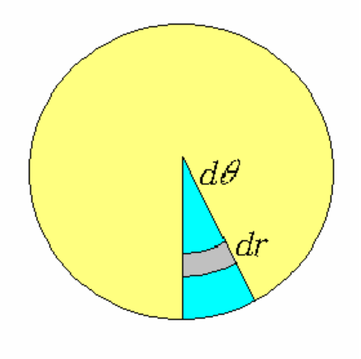
- 단면 극관성 모멘트(polar moment of inertia of cross-sectional area): I_p

$$I_p = \int r^2 dA$$

ex) 반경 R(직경 d)인 원통단면의 경우

$$I = \int_0^{2\pi}\int_0^R (r^2)(r\,d\Theta)dr = \int_0^{2\pi}\int_0^R r^3 dr d\Theta$$

$$= \int_0^{2\pi}\frac{R^4}{4}d\Theta = \frac{\pi R^4}{2} = \frac{\pi d^4}{32}$$



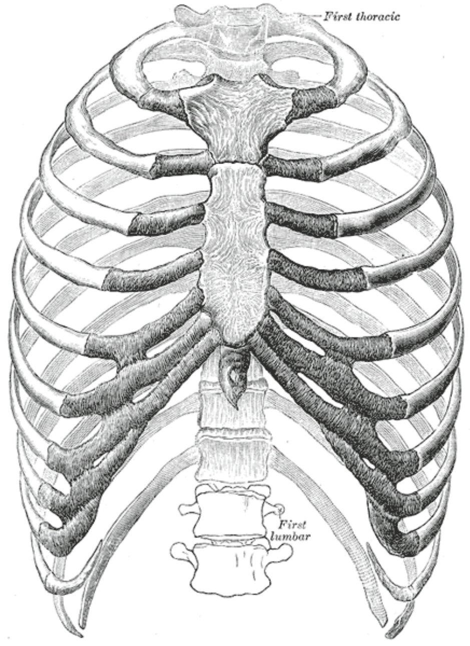
연성 최소화와 단면이용률

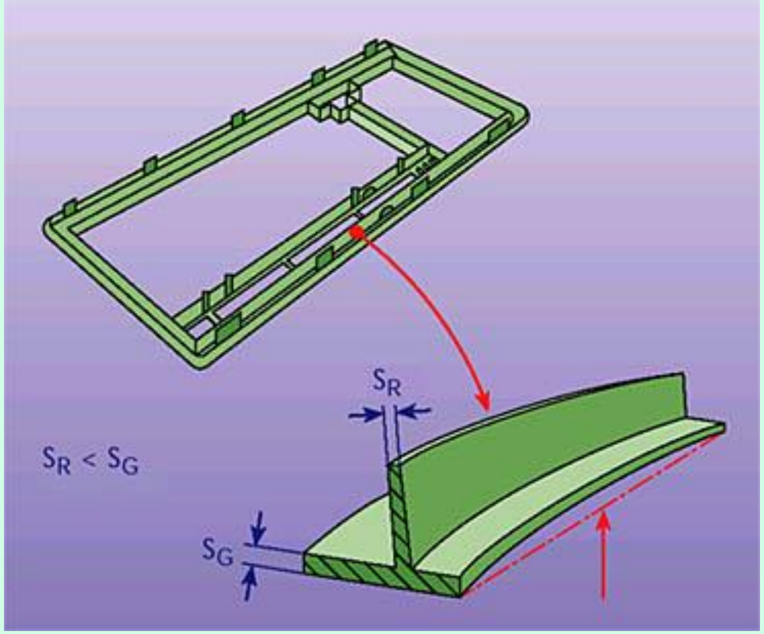
❑ **연성감소 대책 : “Rib의 사용을 통한 단면 이용률의 증대”**

➢ 경사진 rib를 사용할 경우 효과적

⇒ 비틀림하중은 rib부에서 굽힘하중으로 변환

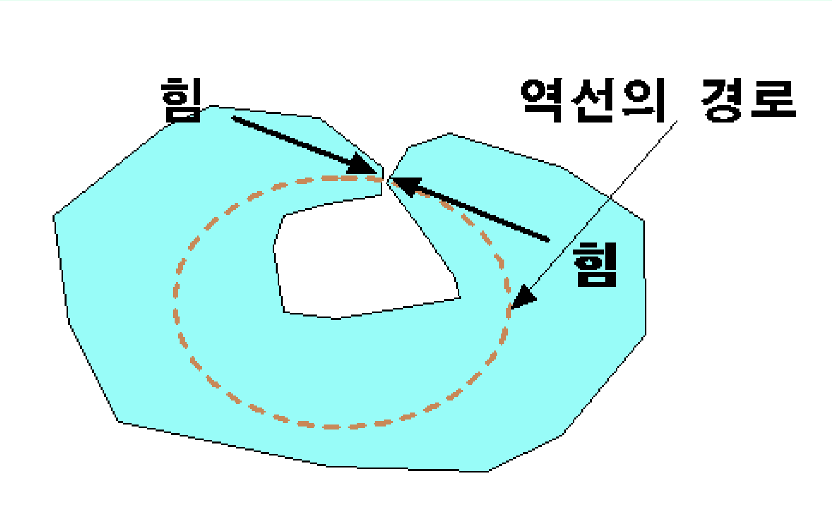
⇒ 극관성 모멘트의 향상 효과



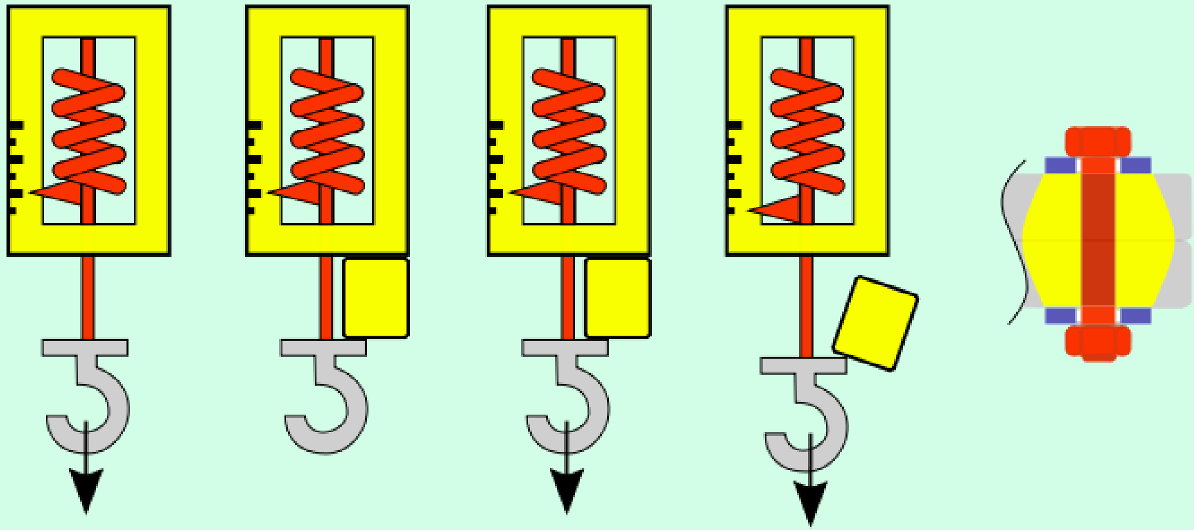


연성의 최소화와 역선의 최소화

- ❑ 연성감소 대책 : “역선의 최소화”
  - ➢ 역선의 길이가 짧으면 굽힘변형이 작게 되는 경우가 일반적
  - ➢ 역선의 길이가 길면 역선의 경로를 이루는 요소들의 변형이 누적
    ⇒ 전체 변형이 증가
  - ➢ 역선은 짧게, 역선 내의 요소들의 연성은 가능한 한 작게 설계



연성 최소화와 예압



Pre-load = 1000 lbf

A 1000 lbf pre-load is applied to a scale.

External load removed

A block is insterted and the load is removed. The spring scale is unaffected.

External load = 0 - 1000 lbf

Any load may be applied, up to the pre-load, and the spring scale will not move, as long as the block is stiff.

External load > 1000 lbf

Only when the external load exceeds the pre-load does the spring scale move.

This analogy may be applied to bolted joints when the members being clamped are much more stiff than the bolt.

연성 최소화와 예압

❑ 연성감소 대책 : "예압의 부여"

- ➢구름 안내의 경우 양방향에서 예압량 부여

  ⇒ 예압량 x_0의 부여

- ➢ 외력에 의하여 x 만큼 변형된다면

$$F_1 = F_a + F_2 \quad \text{from} \quad \sum F_y = 0$$

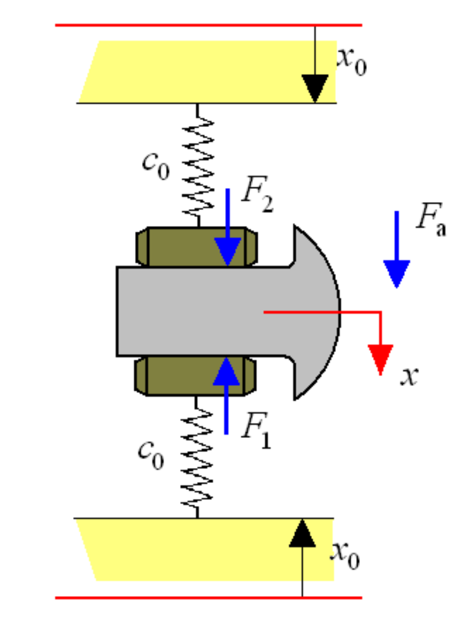
- ➢ 상당연성 c는 x만큼 변형하였으므로

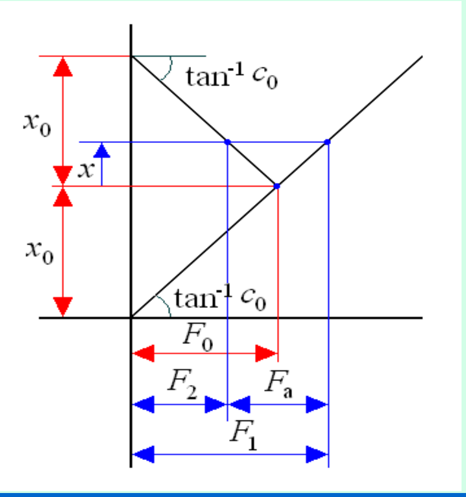
$$c = \frac{x}{F_a} = \left(\frac{c_o F_a}{2}\right) / F_a = \frac{c_o}{2}$$

  ⇒ 연성이 반으로 감소

- ➢ 예압을 줄 경우 하중을 지지하는

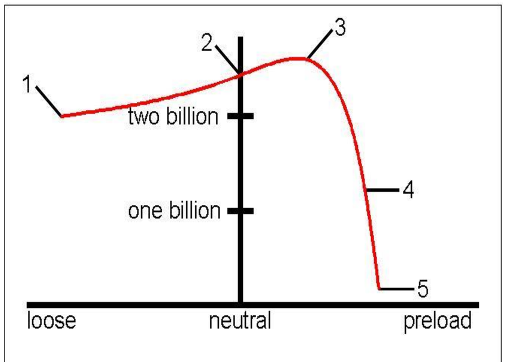
  볼의 개수 증가로 수명 연장의 효과



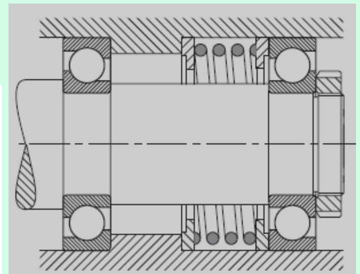


베어링 예압의 효과

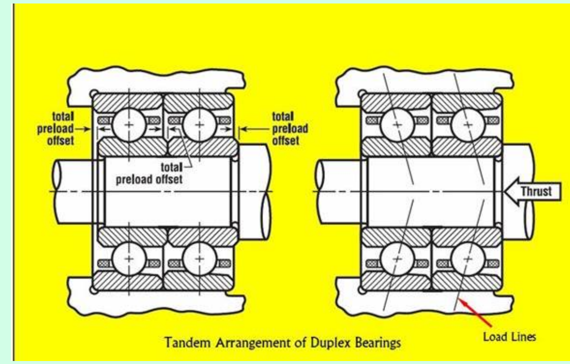
Radial

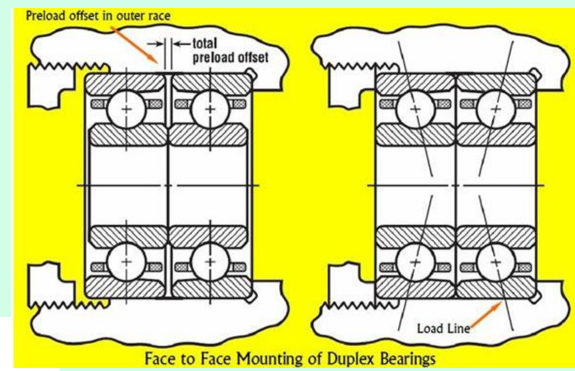


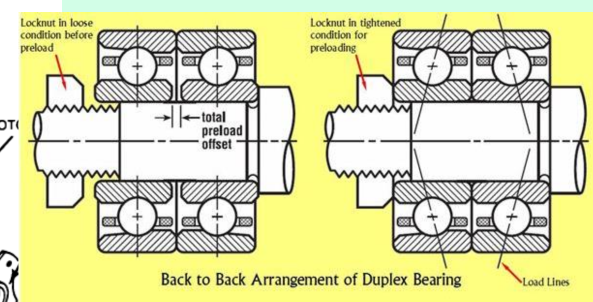
Bearing preload vs. life in miles



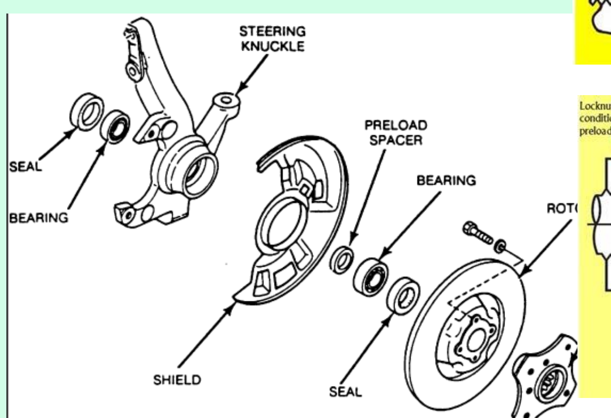
Duplex





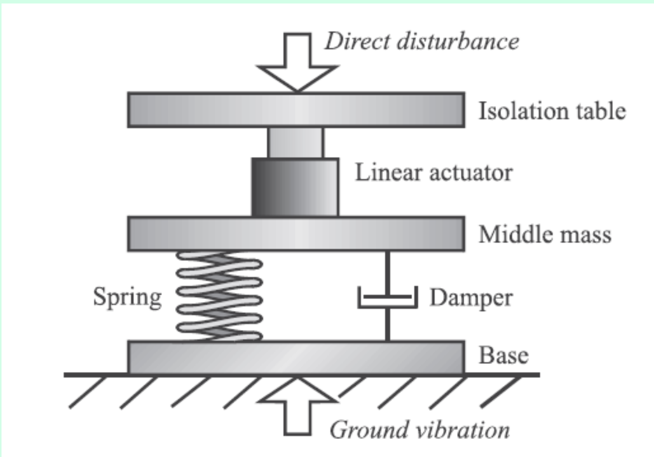


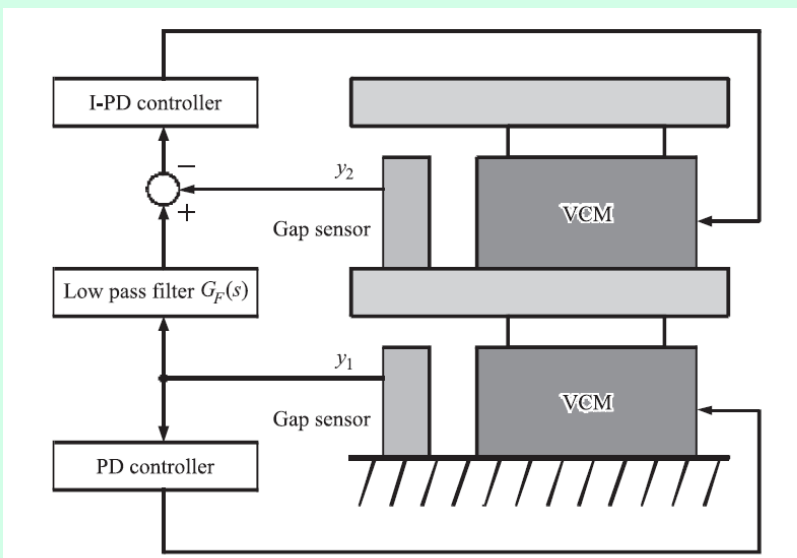
Thrust

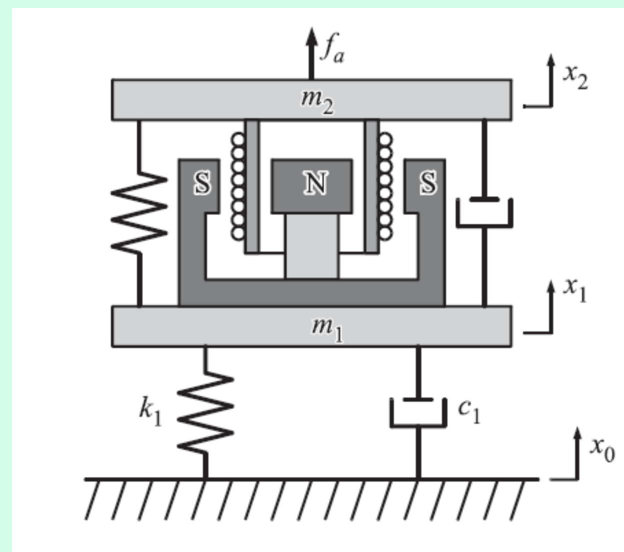


베어링 예압의 효과

Zero compliance realization



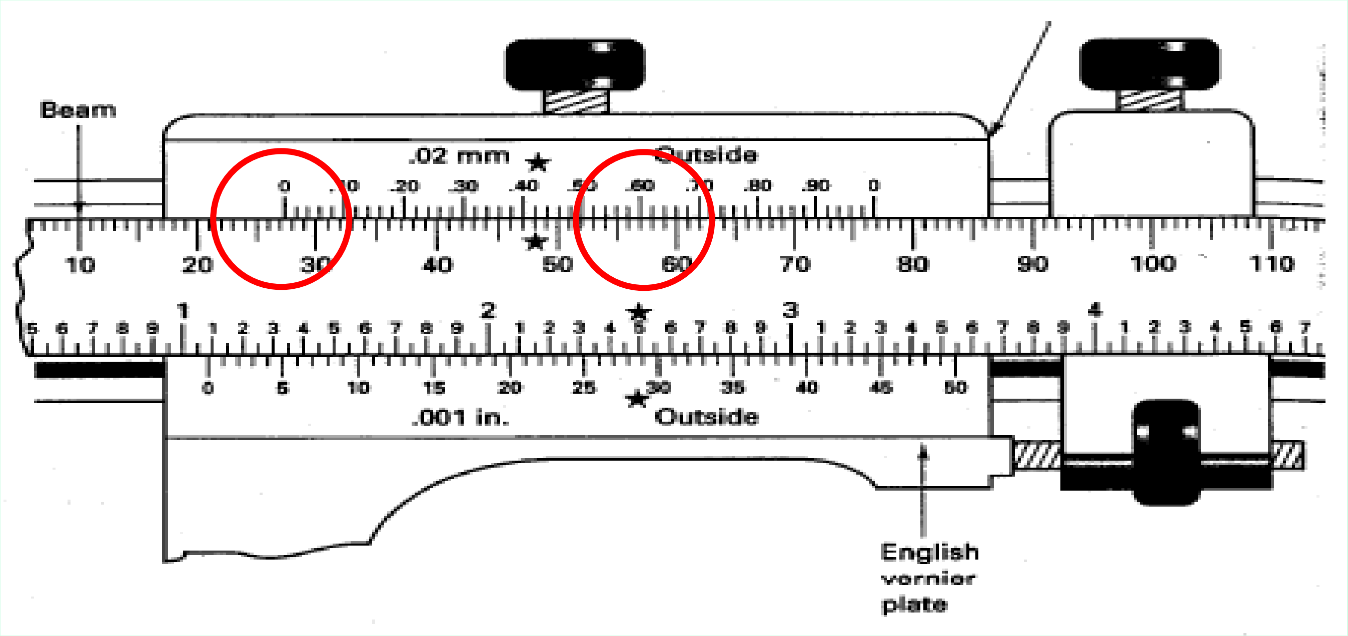




Basic model of suspension with a VCM

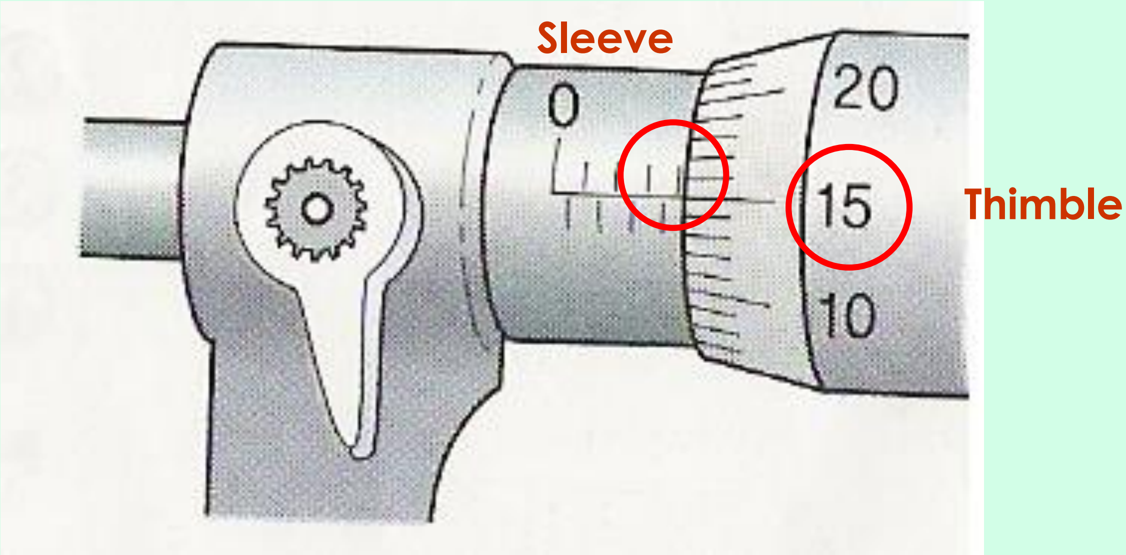
The end of Ch. 8

버어니어 캘리퍼스 읽는법



- ❑ Zero mark is between 27 and 28 mm
- ❑ Look along the graduated scale to two marks that line up (become collinear)
- ❑ Read from ‘outside’ scale - 0.60
- ❑ Reading is 27+0.60 = 27.60mm

마이크로 미터 읽는법



- ❑ Read the total sleeve marks : 8
  - ➔ 0.5mm X 8 = 4mm
- ❑ Read the thimble's mark that lines up with the sleeve line : 15
  - ➔ 0.01mm (=1/100) X 15 = 0.15mm
- ❑ Reading is 4+0.15 = 4.15mm

상용재료의 영률, 전단탄성률

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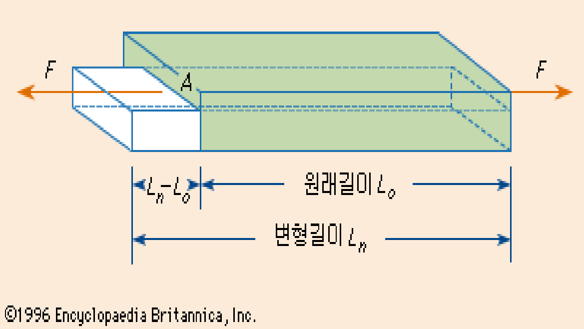

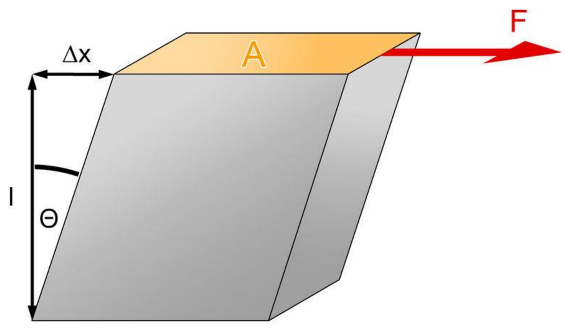

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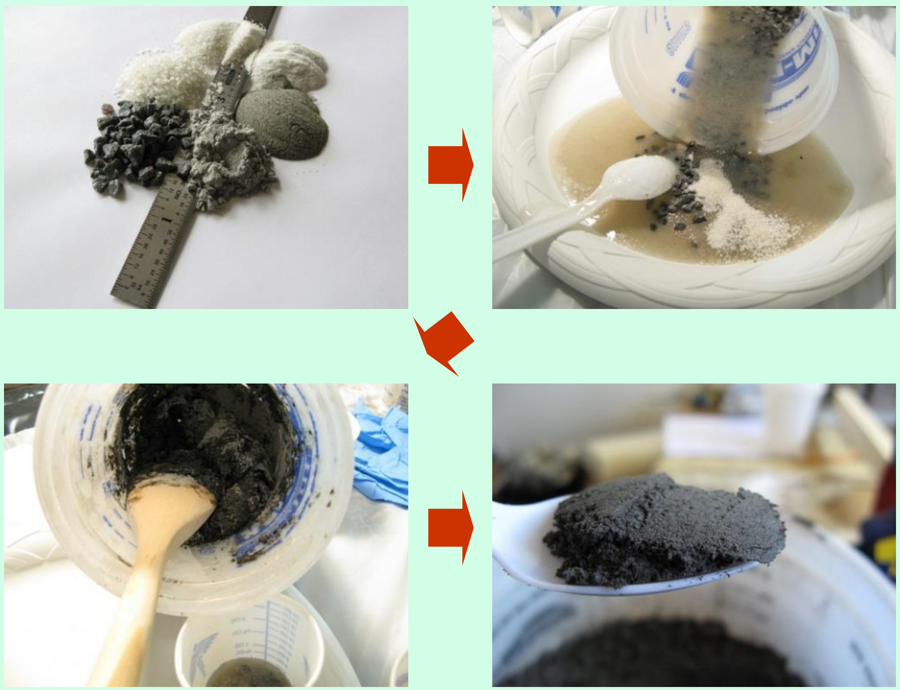
Epoxy-granite (polymer concrete)

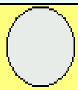
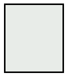
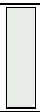
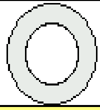
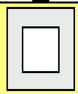
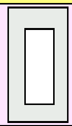
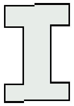
- Epoxy granite can replace cast iron and steel for superior vibration damping, better performance, longer tool life and lower assembly cost.

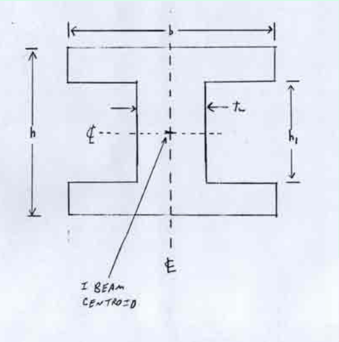


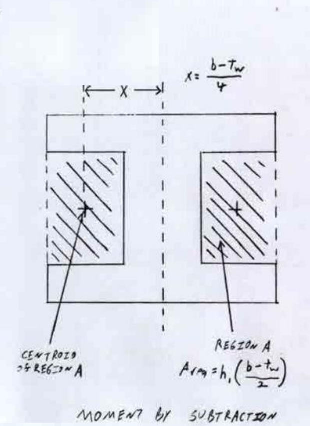
Mixing Procedure

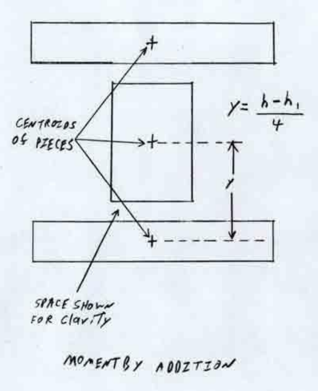
Casted Slider

단면 2차 모멘트(2nd moment of area, area moment of inertia, 2nd moment of inertia)

형상	단면 2차 모멘트	면적	재료의 소비율
	$\pi d^4/64$ 1017.36	d=12 113.04	100
	$h^4/12$ 833.3	h=10 100	88.5
	$bh^3/12$ 1687.5	h=15, b=6 90	79.6
	$\pi(d_2^4-d_1^4)/64$ 1993.2	d_2 =15, d_1=10 98.125	86.8
	$(h_2^4 - h_1^4)/12$ 2368	h_2=14, h_1=10 96	84.9
	$(b_2h_2^3 - b_1h_1^3)/12$ 4174	h_2 = 21, b_2 =7 h_1=17 b_1 = 3 96	84.9
	$[ad^3-h^3(a-t)]/12$ 2830	a=10, d=16, h=10, t=3 90	79.6



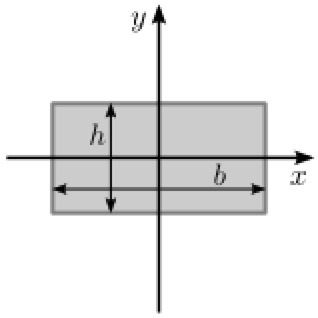




- A property of a shape that is used to predict its resistance to bending and deflection.
- It is not the same thing as the moment of inertia, which is used to calculate angular acceleration.

$$I_x = \iint_R y^2\, dA = \int_{-b/2}^{b/2}\int_{-h/2}^{h/2} y^2\, dy\, dx = \int_{-b/2}^{b/2} \frac{1}{3}\frac{h^3}{4}\, dx = \frac{bh^3}{12}$$

$$I_y = \iint_R x^2\, dA = \int_{-b/2}^{b/2}\int_{-h/2}^{h/2} x^2\, dy\, dx = \int_{-b/2}^{b/2} hx^2\, dx = \frac{b^3h}{12}$$



Using the perpendicular axis theorem we get the value of J_z.

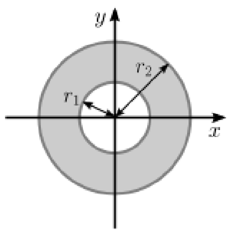
$$J_z = I_x + I_y = \frac{bh^3}{12} + \frac{hb^3}{12} = \frac{bh}{12}(b^2 + h^2)$$

$$I_{x,circle} = \iint_R y^2\, dA = \iint_R (r\sin\theta)^2\, dA = \int_0^{2\pi}\int_0^r (r\sin\theta)^2\,(r\, dr\, d\theta) =$$

$$= \int_0^{2\pi}\int_0^r r^3 \sin^2\theta\, dr\, d\theta = \int_0^{2\pi} \frac{r^4\sin^2\theta}{4}\, d\theta = \frac{\pi}{4}r^4$$

$$J_{z,circle} = \iint_R r^2\, dA = \int_0^{2\pi}\int_0^r r^2\,(r\, dr\, d\theta) = \int_0^{2\pi}\int_0^r r^3\, dr\, d\theta =$$

$$= \int_0^{2\pi} \frac{r^4}{4}\, d\theta = \frac{\pi}{2}r^4$$



$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

Now, the polar moment of inertia about the z axis for an annulus is simply, as stated above, the difference of the second moments of area of a circle with radius r_2 and a circle with radius r_1.

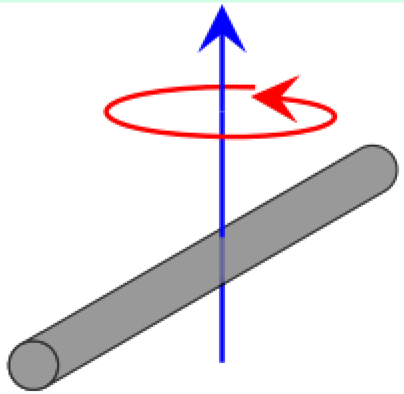
$$J_z = J_{z,r_2} - J_{z,r_1} = \frac{\pi}{2}r_2^4 - \frac{\pi}{2}r_1^4 = \frac{\pi}{2}({r_2}^4 - {r_1}^4)$$

Alternatively, we could change the limits on the dr integral the first time around to reflect the fact that there is a hole. This would be done like this.

$$J_z = \iint_R r^2\, dA = \int_0^{2\pi}\int_{r_1}^{r_2} r^2\,(r\, dr\, d\theta) = \int_0^{2\pi}\int_{r_1}^{r_2} r^3\, dr\, d\theta =$$

$$= \int_0^{2\pi}\left[\frac{r_2^4}{4} - \frac{r_1^4}{4}\right] d\theta = \frac{\pi}{2}\left(r_2^4 - r_1^4\right)$$

Example calculation of moment of inertia



$$I_{C,\mathrm{rod}} = \iiint_Q \rho\, x^2\, dV = \int_{-\ell/2}^{\ell/2} \rho\, x^2\, s\, dx = \rho s \frac{x^3}{3}\Big|_{-\ell/2}^{\ell/2} = \frac{\rho s}{3}\left(\frac{\ell^3}{8} + \frac{\ell^3}{8}\right) = \frac{m\ell^2}{12},$$

where $m = \rho s \ell$ is the mass of the rod.

- The moment of inertia of a **thin disc**

$$I_{C,\mathrm{disc}} = \iiint \rho\, r^2\, dV = \int_0^{2\pi} \int_0^R \rho r^2 s r\, dr\, d\theta = 2\pi\rho s \frac{R^4}{4} = \frac{mR^2}{2},$$

where $m = \pi R^2 \rho s$ is its mass.

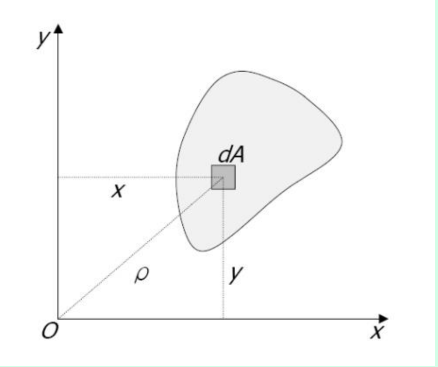
- The moment of inertia of the compound pendulum

$$I_P = I_{C,\mathrm{rod}} + M_{\mathrm{rod}}\left(\frac{L}{2}\right)^2 + I_{C,\mathrm{disc}} + M_{\mathrm{disc}}(L+R)^2,$$

단면 극관성 모멘트(Polar moment of inertia)

형상	단면 극관성 모멘트	면적	재료의 소비율
(circle)	$\pi d^4/32$ 2034.72	d=12 113.04	100
(square)	$0.1406h^4$ 1406	h=10 100	88.5
(hollow circle)	$\pi(d_2^4 - d_1)^4/32$ 3986.33	d_2 =15, d_1=10 98.125	86.8
(rectangle)	khb^3 1447.24	h=15, b=7.5 112.5	99.5

h/b	1	1.1	1.2	1.5	2.0	2.5	3.0	4.0	6.0	8.0	10.0	∞
k	0.1406	0.1540	0.1661	0.1958	0.2287	0.2494	0.2633	0.2808	0.2983	0.3071	0.3123	0.3333

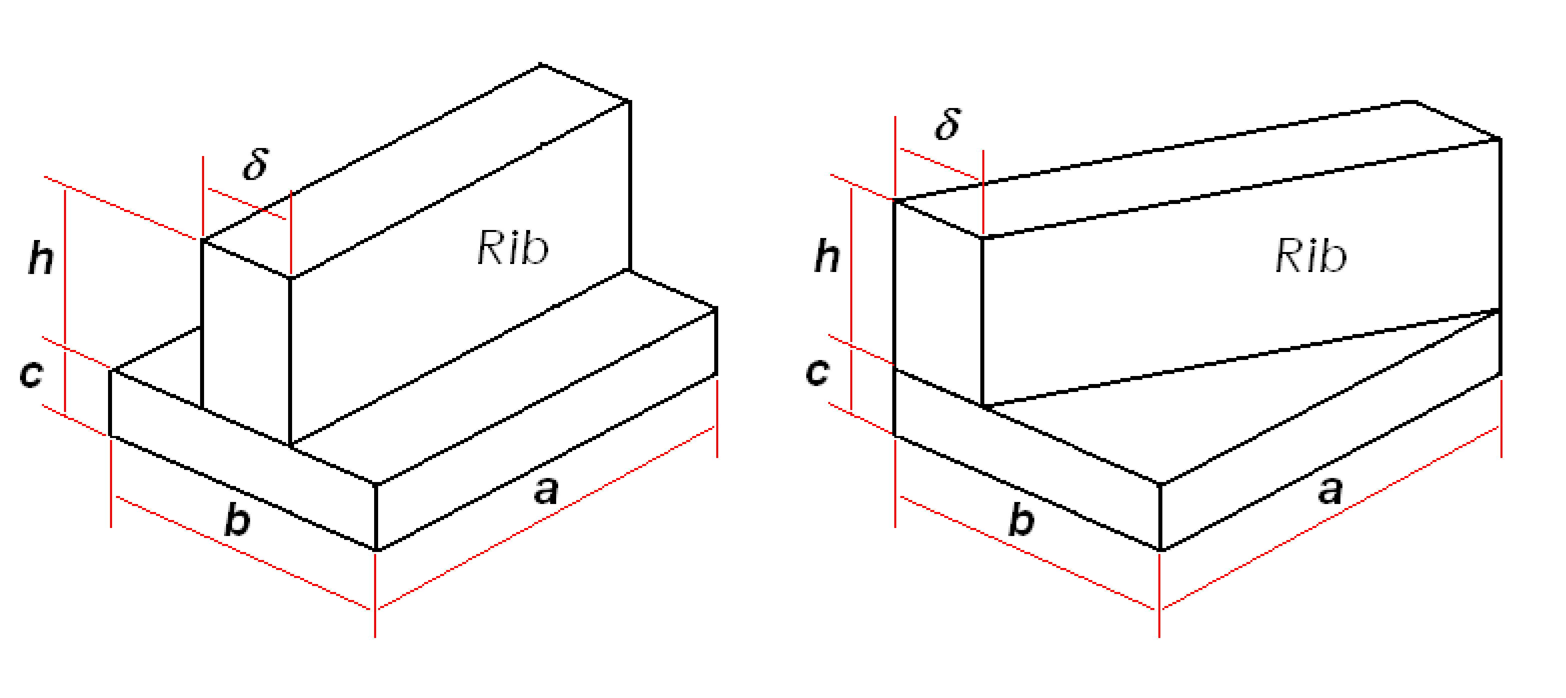


$$J_x = \int r^2 \, dA$$

- J_x = the polar moment of inertia about the axis x
- dA = an elemental area
- r = the radial distance to the element dA from the axis x

- A quantity used to predict an object's ability to resist torsion.
- It is not the same thing as the moment of inertia, which is used to calculate angular acceleration.

리브에 의한 단면이용률 향상



$I_p = bc^3/3 + h\delta^3/3$

$I_p = bc^3/3 + h^3\delta/3$