Advanced Solid Modeling

Basic of CAE

DEPARTMENT OF
MECHATRONICS ENGINEERING

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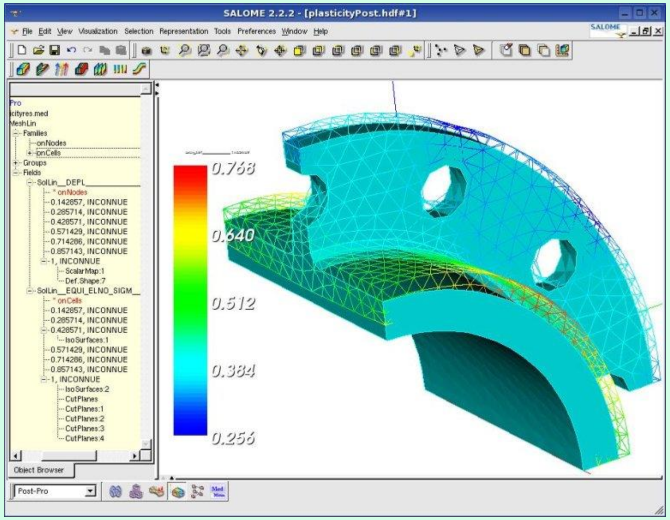
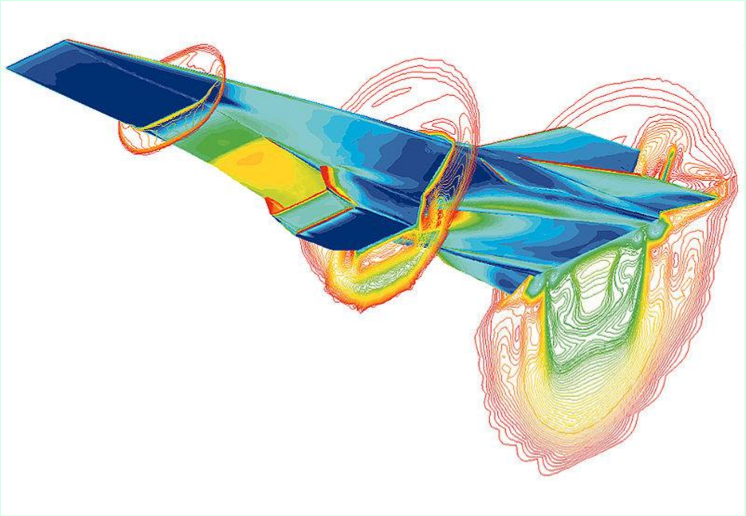
What is CAE?

- Computer-aided engineering (CAE) is the broad usage of computer software to aid in engineering analysis tasks. It includes <u>**Finite Element Analysis (FEA)**</u>, <u>**Computational Fluid Dynamics (CFD)**</u>, Multibody dynamics (MBD), and optimization.





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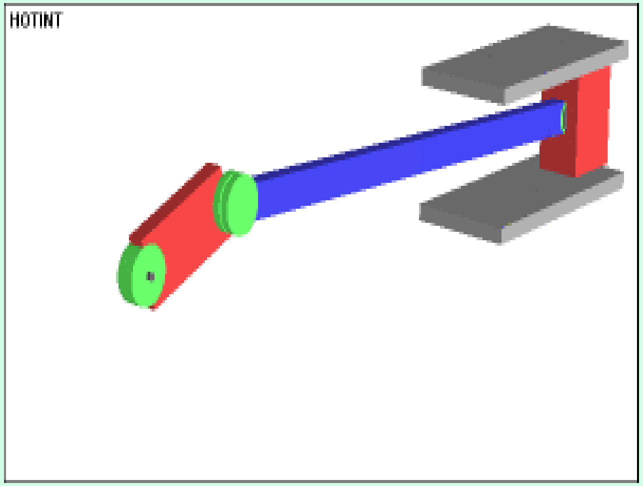
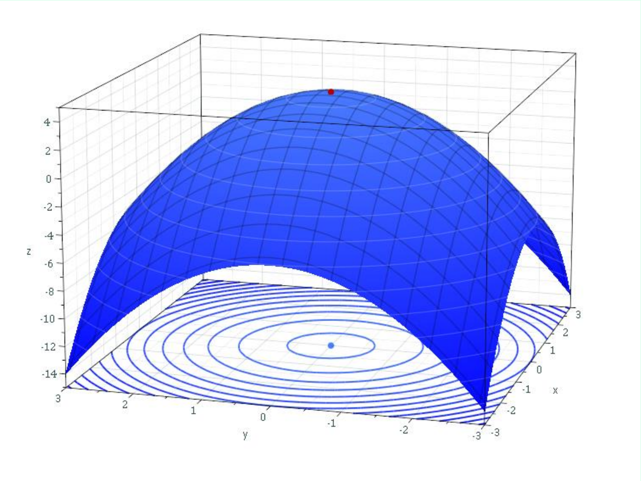
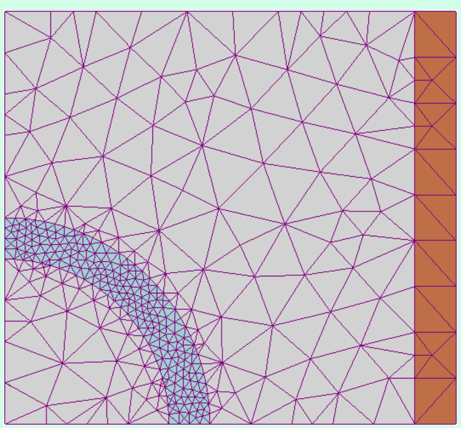




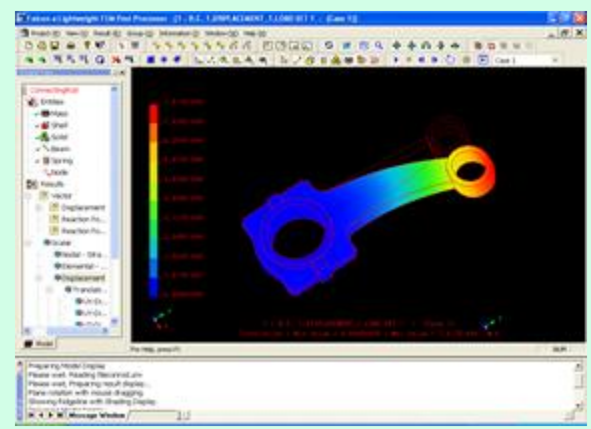
What is CAE?

- In general, there are three phases in any computer-aided engineering task:
  - Pre-processing – defining the model and environmental factors to be applied to it. (typically a finite element model, but facet, voxel and thin sheet methods are also used)
  - Analysis solver (usually performed on high powered computers)
  - Post-processing of results (using visualization tools)



Pre-processing

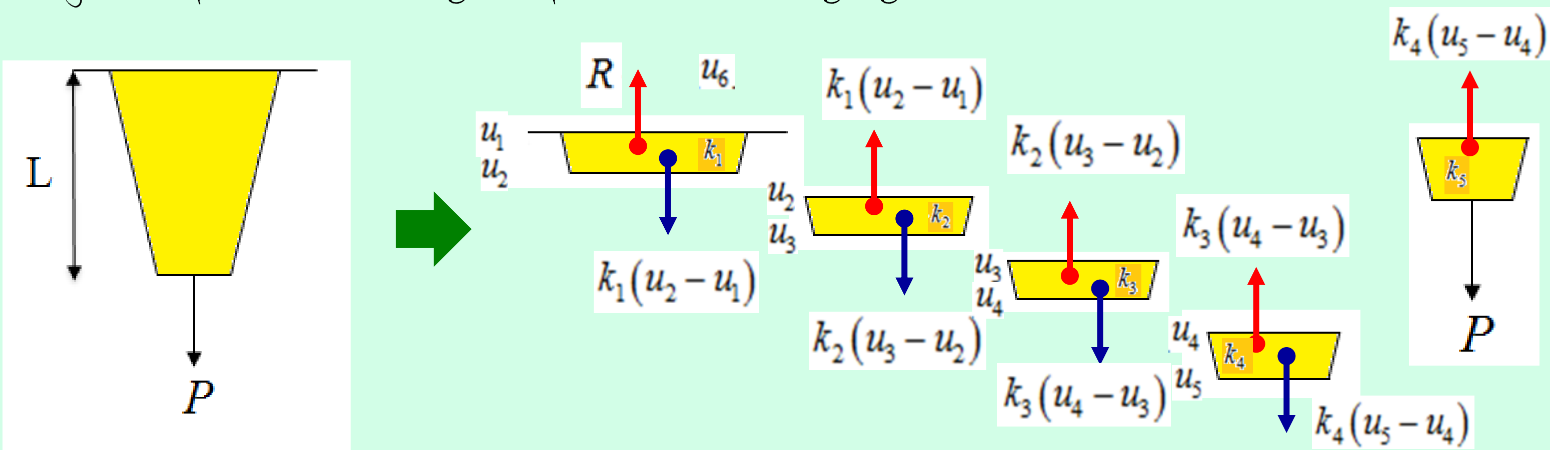
Analysis solver



Post-processing

CAE basic theory

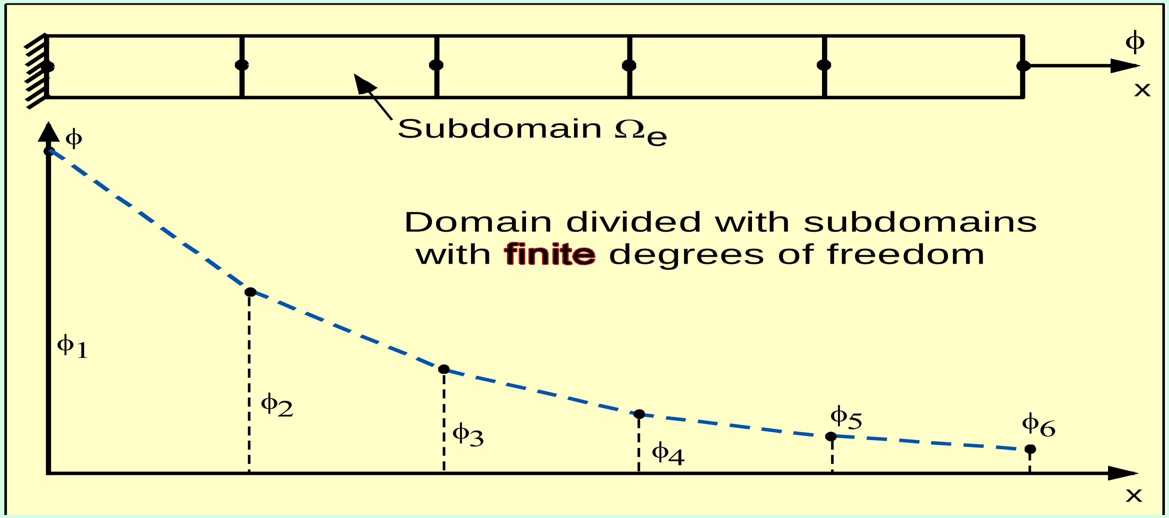
- FEM(Finite Element Method)/FEA(FE Analysis)
  - A numerical technique for finding approximate solutions to boundary value problems for differential equations.
  - Example: Deformation of a bar with a non-uniform circular cross section subject a force P. (Weight of the bar is negligible).



$$
\begin{aligned}
&R-k_1(u_2-u_1)=0\\
&k_1(u_2-u_1)-k_2(u_3-u_2)=0\\
&k_2(u_3-u_2)-k_3(u_4-u_3)=0\\
&k_3(u_4-u_3)-k_4(u_5-u_4)=0\\
&k_4(u_5-u_4)-P=0
\end{aligned}
\Longrightarrow
\begin{bmatrix}
k_1 & -k_1 & 0 & 0 & 0 & 0\\
-k_1 & k_1+k_2 & -k_2 & 0 & 0 & 0\\
0 & -k_2 & k_2+k_3 & -k_3 & 0 & 0\\
0 & 0 & -k_3 & k_3+k_4 & -k_4 & 0\\
0 & 0 & 0 & -k_4 & k_4+k_5 & -k_5\\
0 & 0 & 0 & 0 & -k_5 & k_5
\end{bmatrix}
\begin{Bmatrix} u_1\\ u_2\\ u_3\\ u_4\\ u_5\\ u_6 \end{Bmatrix}
=
\begin{Bmatrix} -R\\ 0\\ 0\\ 0\\ 0\\ P \end{Bmatrix}
\Longrightarrow
[\mathbf{K}]\{\mathbf{u}\}=\{\mathbf{P}\}-\{\mathbf{R}\}
$$

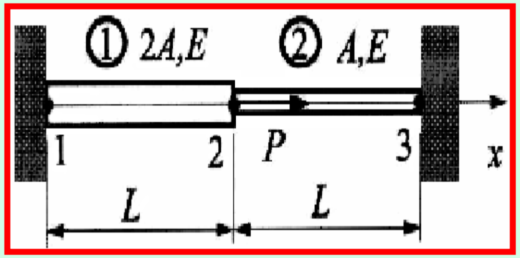
CAE basic theory

- FEM(Finite Element Method)/FEA(FE Analysis)
  - View the problem domain as a collection of subdomains (elements)
  - Solve the problem at each subdomain
  - Assemble elements to find the global solution
  - Solution is guaranteed to converge to the correct solution if proper theory, element formulation and solution procedure are followed

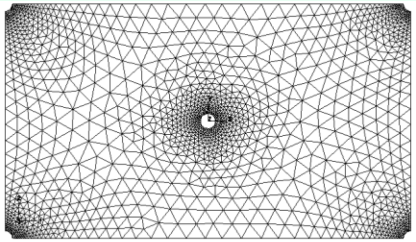


CAE basic theory

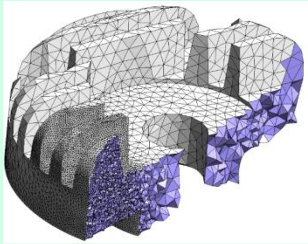
- FEM general steps
  - Discretize & Select the Element Types



1D



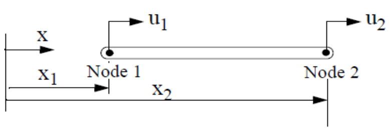
2D



3D

  - Select a Displacement Function

Linear Approximation: $u(x) = C_1 + C_2 x$

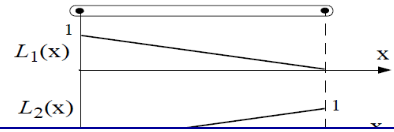


Conditions: $u(x_1) = u_1 \qquad u(x_2) = u_2$

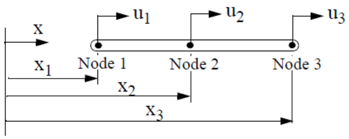
$$u(x) = u_1\left(\frac{x - x_2}{x_1 - x_2}\right) + u_2\left(\frac{x - x_1}{x_2 - x_1}\right)$$

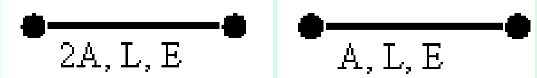
Define: $L_1(x) = \left(\frac{x - x_2}{x_1 - x_2}\right) \qquad L_2(x) = \left(\frac{x - x_1}{x_2 - x_1}\right)$

$$u(x) = u_1 L_1(x) + u_2 L_2(x) = \sum_{i=1}^{2} u_i L_i(x)$$



Quadratic displacement function: $u(x) = C_1 + C_2 x + C_3 x^2$

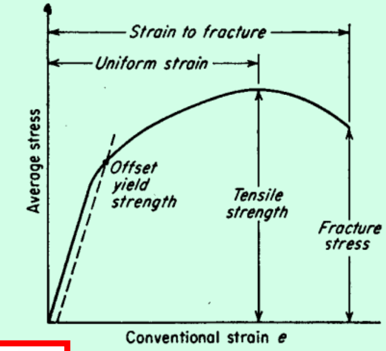




$U(x) = \alpha_1 + \alpha_2 \cdot x \qquad U(x) = \alpha_2 + \alpha_3 \cdot x$

CAE basic theory

- FEM general steps
  - Define the Stress/Strain Relationships
  - Derive the Element Stiffness Matrix & Equations



$$\varepsilon_x = \frac{d}{dx} u \qquad \sigma_x = E \cdot \varepsilon_x$$

Element 1,

$$\mathbf{k}_1 = \frac{2EA}{L} \begin{array}{c} \begin{array}{cc} u_1 & u_2 \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \end{array}$$

Element 2,

$$\mathbf{k}_2 = \frac{EA}{L} \begin{array}{c} \begin{array}{cc} u_2 & u_3 \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array} \begin{array}{c} 2 \\ 3 \end{array}$$

See proof!

  - Assemble the Element Equations to Obtain the Global & Introduce Boundary Conditions: {F} = [K]{D} (see Previous slide!)

$$[K] = \frac{EA}{L} \begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

  - Solve for the Unknown Degrees of Freedom
  - Solve for the Element Strains & Stresses
  - Interpret the Results

Load and boundary conditions (BC)

$$u_1 = u_3 = 0, \qquad F_2 = P$$

$$\frac{EA}{L}[3]\{u_2\} = \{P\} \qquad u_2 = \frac{PL}{3EA} \qquad \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \frac{PL}{3EA} \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

Proof

$$\sigma_x = E \cdot \varepsilon_x$$

$$\sigma_x = \frac{P}{A} \qquad E = \text{Youngs Modulus} \qquad \varepsilon_x = \frac{d}{dx}u$$

$$P = A \cdot E \cdot \frac{d}{dx}u$$

When viewing from u1 to u2

$$\frac{d}{dx}u = u_1 - u_2$$

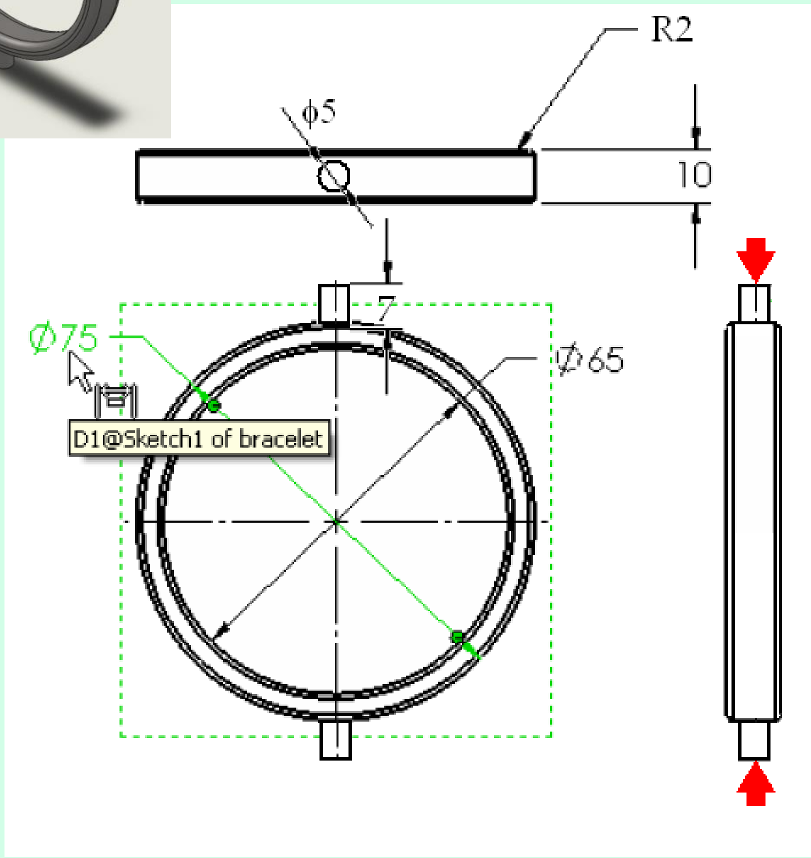
When viewing from u2 to u1

$$\frac{d}{dx}u = u_2 - u_1$$

When combining the two together for the one element you obtain the stiffness matrix

$$\mathbf{k}_1 = \frac{EA}{L} \begin{array}{cc} u_1 & u_2 \\ \left[\begin{array}{cc} 1 & -1 \\ -1 & 1 \end{array}\right] & \begin{array}{c} 1 \\ 2 \end{array} \end{array}$$

CAE role (Teach & Playback)



그림과 같은 링 부재(Alloy steel)의 상하 돌출부에 각각 힘 2kN이 작용할 때 솔리드웍스를 이용하여 정적해석 수행

- 안전계수를 3으로 할 때의 불안정영역
- 피로해석 방법
- 2kN의 힘을 가하였을 때 링이 탄성역에서 정적안정성을 가지도록 설계 (단 링의 외경과 내경 및 돌출부의 직경은 변화시킬 수 없음)
- 모드 해석 실행